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APPLICATIONS OF THE ELECTROLYTIC METHOD—II. MASS TRANSFER WITHIN A TUBE FOR STEADY, OSCILLATING AND PULSATING FLOWS

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Abstract—The electrolytic method (see proceeding article) was used to measure mass transfer in a hydraulic fully-developed flow within a tube for steady, pulsating, and oscillating flows. The mass transfer was investigated at the entrance region as well as after the concentration profile has developed. The mass transfer increases by oscillation up to 2.5 times the value of the steady flow and was up to 12% larger than could be expected according to the "quasi steady model". The recordings show details of turbulence, separation of flow, and backflow near the wall.

NOMENCLATURE

Dimensionless numbers

Nu,	$= \frac{\alpha d}{\lambda}$, Nusselt number;
Pr,	$= \frac{\eta c_p}{\lambda}$, Prandtl number;
Re_S,	$= \frac{d v_S}{\nu}$, Reynolds number of stationary flow;
Re_O,	$= \frac{d v_O}{\nu}$, Reynolds number of oscillating flow;
Sc,	$= \nu/D$, Schmidt number;
Sh,	$= \frac{\beta d}{D}$, Sherwood number;
Sk,	$= \frac{d^2}{\nu t_0}$, Stokes number;
L^+,	$= L\frac{v^*}{\nu}$, dimensionless length in the direction of flow;
β^+,	$= \beta/v^*$, dimensionless mass-transfer coefficient;
c_p,	specific heat capacity [J kg^{-1} K^{-1}];
D,	diffusion coefficient [m^2 s^{-1}];
d,	diameter of the test section [m];
L,	distance from the beginning of mass transfer to the end of the cathode [m];
l,	length of the cathode [m];
t,	time [s];
t_0,	duration of a period [s];
v_0,	amplitude of the oscillation of flow [m s^{-1}];
v_S,	stationary velocity [m s^{-1}];
v^*,	$= (\tau_w/\rho)^{1/2}$, shear stress velocity.

Greek symbols

α,	heat-transfer coefficient [Wm^{-2} K^{-1}];
β,	mass-transfer coefficient [m s^{-1}];
η,	dynamic viscosity [kg m^{-1} s^{-1}];
λ,	thermal conductivity [W m^{-1} K^{-1}];
ν,	kinematic viscosity [m^2 s^{-1}];
ρ,	density [kg m^{-3}];
τ_w,	shear stress at the wall [kg m^{-1} s^{-2}];
ω,	angular velocity [s^{-1}].

Superscripts and mathematical symbols

L,	local value at L;
$O \rightarrow L$,	average from beginning of mass transfer to L;
P,	pulsation;
q,	quasi-steady;
S,	steady,
∞,	developed concentration profile;
$\mid\ \mid$,	absolute value;
$\bar{}$,	averaged over a period.

1. INTRODUCTION

PULSATING and oscillating flows are often used in heat- and mass-transfer equipment especially in chemical engineering. Examples are pulsating columns for extraction and coolers following reciprocating pumps. It is well known, that pulsation increases the transfer coefficients, however, the details of this are not yet known. The electrolytic method allows measurement of the instantaneous value of mass transfer for the different phases of a pulsating flow and shows, if there is turbulence, separation of flow from the wall, and backflow. This helps to understand the mechanism of this increase.

2. TEST SECTION

The test section [1] was a tube 346 mm long with an inner diameter of 22 mm. This tube was divided longitudinally into two parts: two thirds of the interior surface for the anode, the other third could be used as cathodes. In this larger cathodes smaller,

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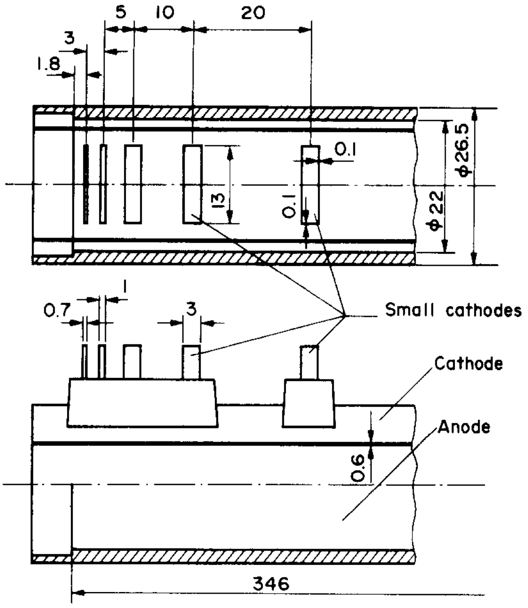

FIG. 1. Test section with the locations of electrodes.

isolated cathodes were inserted (Fig. 1) for measuring local values of mass transfer. Also a part of these electrodes could be used as anodes, e.g. for the measurements of Figs. 2, 5, 6 and 7. Before entering the test section, the electrolyte had to pass a tube, 810 mm long, having the same diameter as the test section. Therefore, the flow entering it, was hydraulically fully developed. The flow was measured using an electromagnetic flow meter. The oscillating or pulsating flow was generated using a bellows pump. It was possible to measure instantaneous values and, by means of an electronical integrator, also values of the current averaged over various lengths of time.

The ranges of the measurements were:

Steady Re number 0 and $100 < Re_S \leqslant 51\,250$; oscillating Re number $0 \leqslant Re_0 \leqslant 42\,800$; Stokes number $18.7 \leqslant Sk \leqslant 600$
(corresponding frequencies lay between 0.069 and $1.74\,\mathrm{s}^{-1}$) Schmidt number 2770 and 11 200.

Dimensionless distances from the beginning of mass transfer:

$$0.0321 < L/d < 15.7.$$

The electrolyte was a solution of $0.025\,\mathrm{kmol\,m^{-3}}$ K_3 Fe $(CN)_6 + 0.025\,\mathrm{kmol\,m^{-3}}$ K_4 Fe $(CN)_6$ with 2.0 respectively—for increasing the Sc number—$5.0\,\mathrm{kmol\,m^3}$ Na OH.

3. ENTRANCE REGION AT STEADY FLOWS

The arrangement of the cathodes allowed to measure the local mass-transfer coefficients of the hydraulically developed steady flows at different distances from the beginning of mass transfer. The results for laminar flow agree perfectly with the theory of Lévêque [2], that is with the equation:

$$Sh_{S(0\to L)} = 1.615(d/L)^{1/3} Re_S^{1/3} Sc^{1/3}. \qquad (1)$$

For turbulent flow and not too small values of d/L the equation derived by Elser [3].

$$Sh_{S(0\to L)} = 0.275(d/L)^{1/3} Re_S^{7/12} Sc^{1/3}, \qquad (2)$$

agrees well with the present experimental results (Figs. 2 and 3). For larger values of L or—more exactly—larger values of the dimensionless length $L\dagger = Lv^*/\nu$ the equations derived by Spalding [4] fit very well with the experimental results (Fig. 3). They also agree very well with the experiments of Schütz [5, 6], obtained by the same method, however with a completely different arrangement.

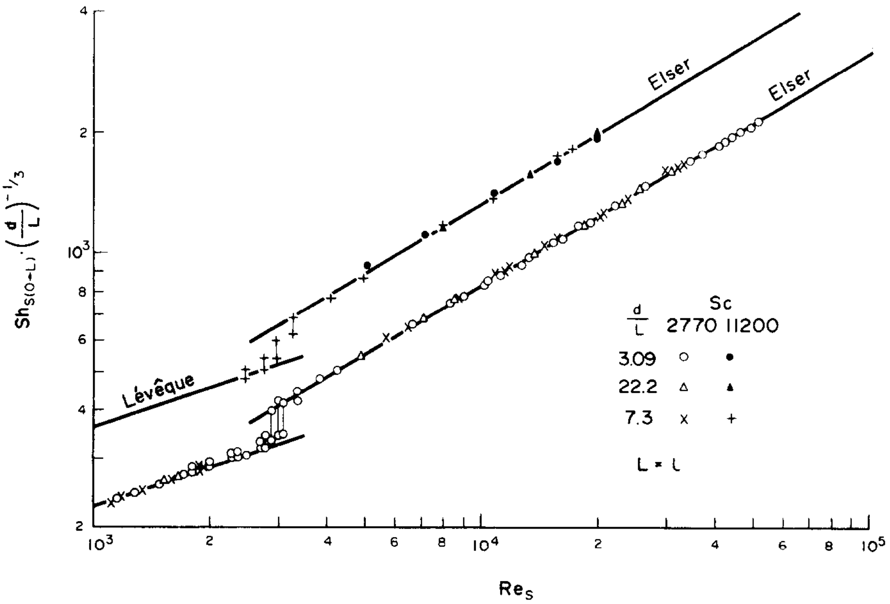

FIG. 2. Average of the Sherwood number from 0 to L for the entry region and turbulent flow.

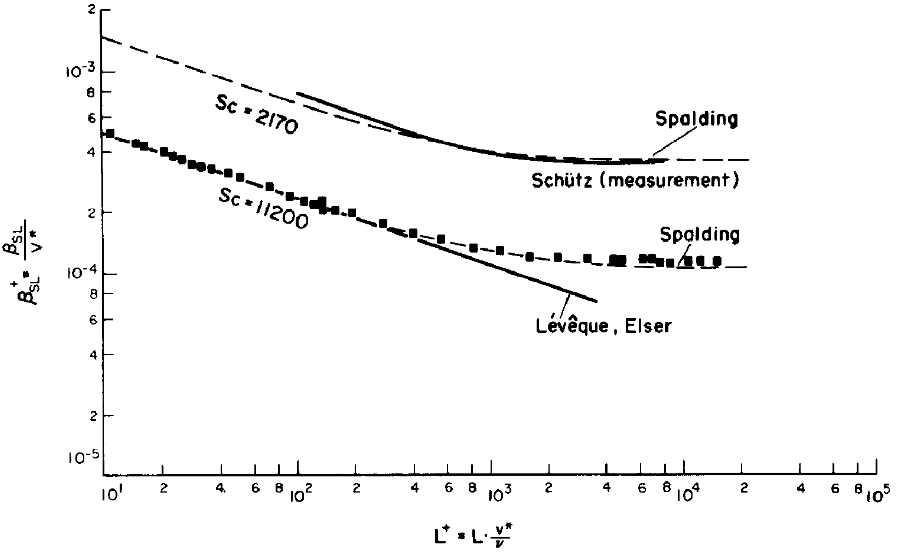

FIG. 3. Local, dimensionless mass-transfer coefficient for turbulent flow as a function of the dimensionless distance from beginning of mass transfer.

4. TURBULENT FLOWS WITH DEVELOPED CONCENTRATION PROFILES

For these measurements all cathodes were connected to voltage. However the current of one of the cathodes only was measured. This cathode was at a mean distance of 183 mm or more downstream from the beginning of mass transfer. The results for both *Sc* numbers—2770 and 11 200—could be correlated with a mean error of the average of 0.12 and 0.33% by the equation:

$$Sh_{S\infty} = 0.02178 Re_S^{7/8} Sc^{0.264}. \qquad (3)$$

The mean errors of the individual measurements are ±1.2% respectively ±2.2%.

5. THE CRITICAL *Re* NUMBER FOR OSCILLATING AND PULSATING FLOW

To demonstrate the influence of the oscillation on the critical *Re* number experiments with oscillating flow were performed. "Oscillating flow" means, that the velocity averaged over a whole period is zero (Fig. 4). In this case the flow is dependent on the Stokes number $Sk = d^2/(\nu t_0)$ and on the amplitude. The dependence Sk_{crit}, that is where turbulence begins, on the Re_0 number gives approximately the following equation:

$$Sk_{\text{crit}} = 8.7 \times 10^{-4} Re_0^{4/3} \qquad \text{for } 2300 < Re_0 < 15\,300. \qquad (4)$$

For larger frequencies, that is for $Sk > 300$ no increase beyond $Re_{\text{crit}} = 15\,300$ could be detected.

For pulsating flows the dependences are more complicated, as a third variable, the *Re* number Re_S of the steady flow, enters the problem [1].

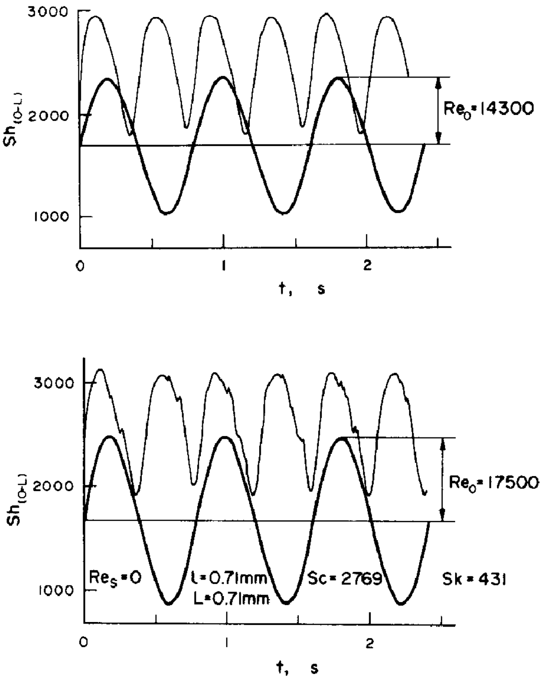

FIG. 4. Reynolds and Sherwood numbers as a function of time in the entry region at beginning of turbulence.

6. MASS TRANSFER FOR PULSATING AND OSCILLATING FLOWS

In the "quasi-steady model" [7] it is assumed, that the instantaneous *Sh* number Sh_{P_q} is equal to that of a steady flow, whose velocity is equal to the instantaneous velocity of the oscillating flow in question. Assuming the *Sh* number being proportional to Re^n it follows:

$$\frac{Sh_{P_q}}{Sh_S} = \frac{1}{2\pi} \int_0^{2\pi} \left[\left(1 + \frac{Re_0}{Re_S} \right) \sin \omega t \right]^n \mathrm{d}(\omega t). \qquad (5)$$

For $n < 1$ this function decreases with increasing Re_0/Re_S. However, the instantaneous *Sh* number cannot become negative. Therefore the absolute value of the integrand has to be taken. Numerical calculations show, that beginning with $Re_0 \approx Re_S$

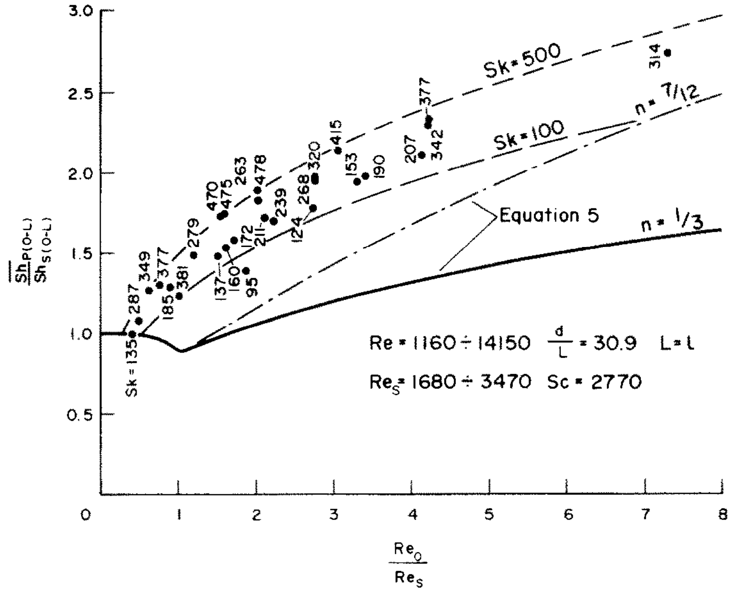

FIG. 5. Quotient of the Sherwood numbers for pulsating and stationary laminar flow averaged from 0 to L as a function of the quotient of the Reynolds numbers.

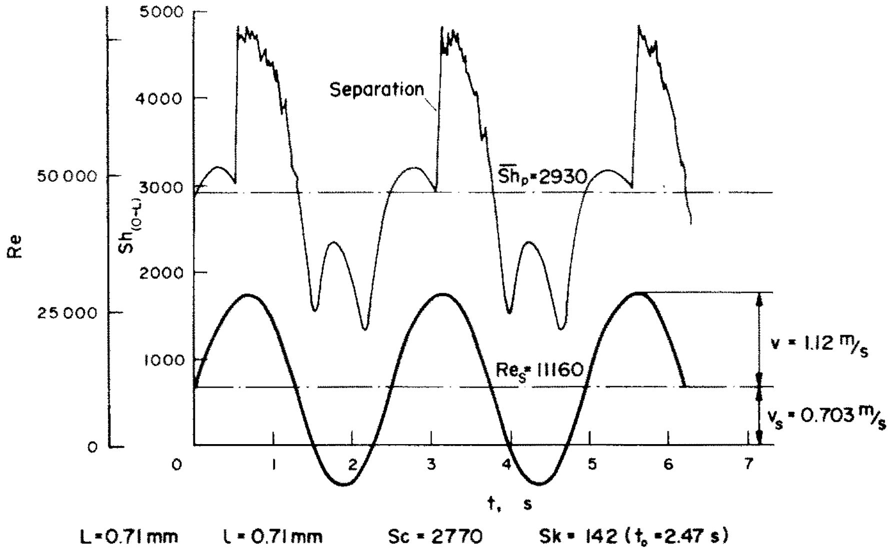

FIG. 6. Reynolds and Sherwood number averaged from 0 to L for pulsating, turbulent flow as a function of time.

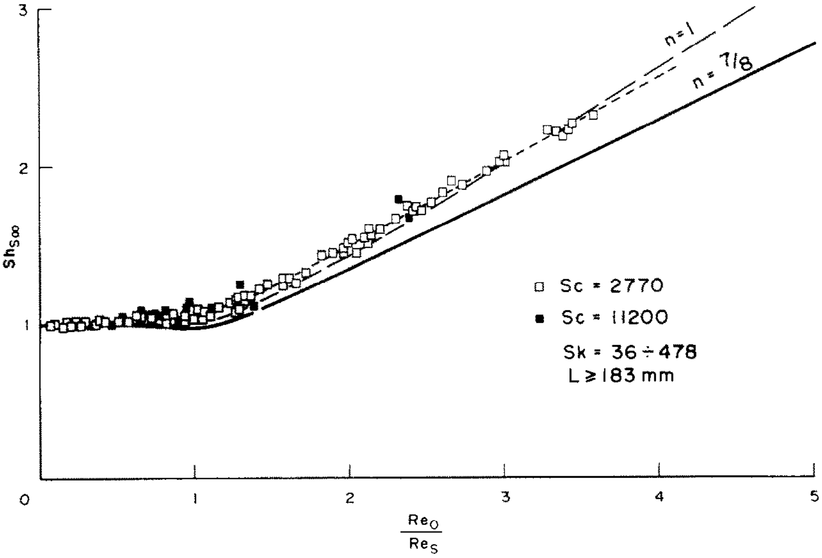

FIG. 7. Quotient of the mean Sherwood numbers for pulsating and stationary turbulent flow as a function of the quotient of the Reynolds numbers for developed concentration profile.

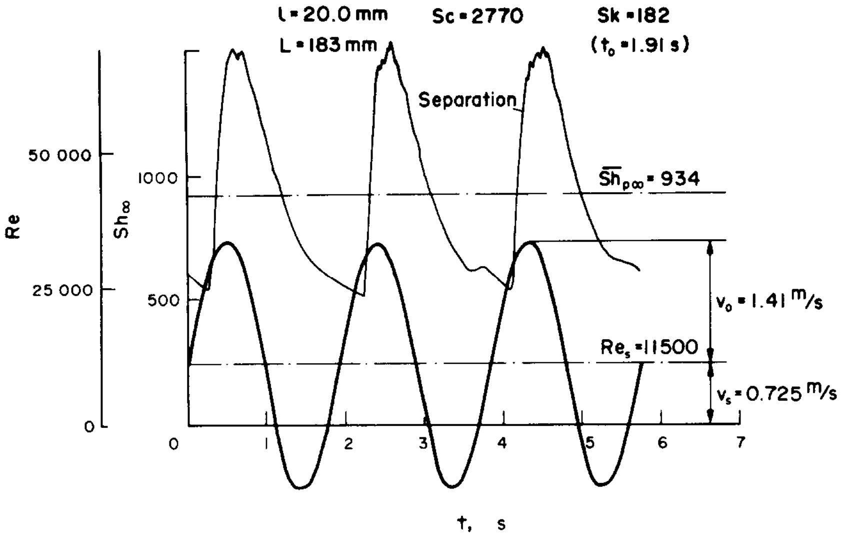

FIG. 8. Reynolds number and mean value of the Sherwood number from $L = 163$ to 183 mm for pulsating, turbulent flow and developed concentration profile as a function of time.

the integral increases with increasing values of Re_0/Re_S.

Figure 5 shows for the entrance region and laminar flow ($n = 1/3$), that depending on the value of Sk the results of the experiments lie above the curves corresponding to the quasi-steady model, equation (5). This may be due to backflow near the wall at large amplitudes. Contrary to former investigations our experimental set up allowed even negative velocities for a fraction of the period. Figure 6 shows such a case. The steep increase of the Sh number, indicating a separation of the flow from the wall and the following turbulence can clearly be seen.

When the concentration profile for turbulent flow is fully developed the measured values lie up to 12% above the curve corresponding to the quasi-steady model (Fig. 7). Within the range of the measurements Sh numbers up to 2.5 times of that of the steady flow were recorded.

Figure 8 shows the instantaneous Sh number for the fully developed flow as a function of time.

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APPLICATIONS DE LA METHODE ELECTROLYTIQUE IIème PARTIE: TRANSFERT MASSIQUE DANS UN TUBE POUR DES ECOULEMENTS PERMANENTS, OSCILLANTS OU PULSES

Résumé—La méthode électrolytique (voir l'article précédent) a été utilisée pour mesurer le transfert massique dans un écoulement hydrauliquement établi dans un tube, pour des écoulements permanents, pulsés et oscillants. Le transfert massique est mesuré dans la région d'entrée aussi bien qu'après que le profil de concentration soit établi. Le transfert massique croît du fait des oscillations jusqu'à 2,5 fois la valeur de l'écoulement permanent et il est jusqu'à 12% plus élevé que ce qui peut être espéré par le modèle quasi permanent. Les enregistrements montrent des détails sur la turbulence, la séparation de l'écoulement et les courants de retour près de la paroi.

ANWENDUNGEN DER ELEKTROLYTISCHEN METHODE TEIL II: STOFFÜBERGANG IN EINEM ROHR BEI STATIONÄREN, OSZILLIERENDEN UND PULSIERENDEN STRÖMUNGEN

Zusammenfassung—Die elektrolytische Methode wird benützt, den Stoffaustausch einer hydraulisch ausgebildeten stationären oder pulsierenden oder oszilierenden Rohrströmung zu untersuchen, und zwar ebensowohl unmittelbar stromab vom Beginn des Stoffaustauschs wie auch bei ausgebildetem Konzentrationsprofil. Innerhalb des Versuchsbereichs stieg der Stoffaustausch bis auf das 2.5 fache des

Wertes für die stationäre Strömung und war bis 12% höher als nach dem quasistationären Model zu erwarten. Die Registrierkurven zeigen Einzelheiten der Turbulenz, der Ablösung und der wandnahen Rückströmung.

ПРИМЕНЕНИЕ ЭЛЕКТРОЛИТИЧЕСКОГО МЕТОДА. ЧАСТЬ 2. ПЕРЕНОС МАССЫ В ТРУБЕ ПРИ СТАЦИОНАРНОМ, КОЛЕБЛЮЩЕМСЯ И ПУЛЬСИРУЮЩЕМ ТЕЧЕНИЯХ

Аннотация — С помощью электролитического метода проведены измерения массопереноса в гидравлически полностью развитом потоке в трубе при стационарном, пульсирующем и колеблющемся течениях. Исследования проводились на входном участке и в области с развитым профилем концентрации. Найдено, что наложение пульсаций интенсифицирует массоперенос примерно в 2,5 раза по сравнению со стационарным течением и что величина диффузионного потока примерно на 12% больше расчётного значения, вычисленного с помощью «квазистационарной модели». Получена детальная картина турбулентного течения, отрыва потока и возвратного течения у стенки.